Constraints on Proton Structure from Precision Atomic-Physics Measurements

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Ground-state hyperfine splittings in hydrogen and muonium are very well measured. Their difference, after correcting for magnetic moment and reduced mass effects, is due solely to proton structure—the large QED contributions for a pointlike nucleus essentially cancel. The rescaled hyperfine difference depends on the Zemach radius, a fundamental measure of the proton, computed as an integral over a product of electric and magnetic proton form factors. The determination of the Zemach radius, (1.019 ± 0.016) fm, from atomic physics tightly constrains fits to accelerator measurements of proton form factors. Conversely, we can use muonium data to extract an experimental value for QED corrections to hydrogenic hyperfine data. There is a significant discrepancy between measurement and theory, in the same direction as a corresponding discrepancy in positronium.

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Introduction.—Quantum electrodynamics (QED) stands out as the most precisely tested component of the standard model. QED predictions for the classic Lamb-shift and hyperfine splittings (HFS) in hydrogen, positronium, and muonium have been confirmed to better than 10 ppm [1,2], 2 ppm [2,3], 2 ppm [1,2], and one part in 1.0×10^7 [1], respectively. The measurements of the electron and positron gyromagnetic ratios agree with order-α^4 perturbative QED predictions to one part in 1.0×10^{11} [4]. QED and gauge theory have thus been validated to extraordinary precision.

In this Letter we shall show how one can combine precision atomic-physics measurements to determine a fundamental property of the proton to remarkable precision. The difference between the ground-state HFS of hydrogen and muonium, after correcting for the different magnetic moments of the muon and the proton and for reduced mass effects, is due to the structure of the proton. The QED contributions for a pointlike nucleus essentially cancel. The largest proton structure contribution to the HFS difference is proportional to the Zemach radius [5,6], which can be computed as an integral over the product of the elastic electric and magnetic form factors of the proton. The remaining proton structure corrections, the polarization contribution [3,7–10] from inelastic states in the spin-dependent virtual Compton amplitude, and the proton size dependence of the relativistic recoil corrections [11,12], have small uncertainties. As we shall show, the resulting high-precision determination of the Zemach radius from the atomic-physics measurements provides an important constraint on fits to accelerator measurements of the proton electric and magnetic form factors.

An important motivation for examining form factor constraints comes from the recent polarization transfer measurements of the proton electric form factor $G_E(Q^2)$ [13–15]. The polarization transfer results are at variance with the published Rosenbluth measurements of G_E. The difference may well be due to corrections from hard two-photon exchange [16,17]. One wants to examine with the maximum possible precision whether the new determinations of $G_E(Q^2)$, falling with respect to $G_M(Q^2)$, is compatible with other information on the form factor. The extraction of the Zemach radius to be described here provides such a constraint.

A sum rule for proton structure.—We now show how one can use the HFS of the muonium atom $(e^- \mu^+)$ to expose the hadronic structure contributions to the hydrogen HFS. For an electron bound to a positively charged particle of mass m_N, magnetic moment $\mu_N = (g_N/2)(e/2m_N)$, and Landé g factor g_N, the leading term in the HFS is the Fermi energy,

$$E_F^N = \frac{8}{3\pi} \alpha^3 \mu_B \mu_N \frac{m_e^3 m_N^3}{(m_N + m_e)^3}. \qquad (1)$$

Here, "N" stands for either the p or μ^+ nucleus. By convention, the exact magnetic moment μ_N is used for the proton or muon, but only the lowest order term, the Bohr magneton μ_B, is inserted for the e^-.

The ground-state hydrogen HFS can be written as

$$E_{\rm HFS}(e^- p) = (1 + \Delta_{\rm QED} + \Delta_R^p + \Delta_S) E_F^p, \qquad (2)$$

where $\Delta_{\rm QED}$ represents QED corrections, Δ_R^p represents recoil effects, including finite-size recoil corrections, and Δ_S represents the proton structure contributions. The corresponding quantity for muonium is simply

$$E_{\rm HFS}(e^- \mu^+) = (1 + \Delta_{\rm QED} + \Delta_R^\mu) E_F^\mu. \qquad (3)$$

We define the fractional difference between the hydrogen and rescaled muonium HFS as

$$\Delta_{\rm HFS} \equiv \frac{E_{\rm HFS}(e^- p)}{E_{\rm HFS}(e^- \mu^+)} \frac{\mu_\mu}{\mu_p} \frac{(1 + m_e/m_p)^3}{(1 + m_e/m_\mu)^3} - 1$$
$$= \frac{E_{\rm HFS}(e^- p)/E_F^p}{E_{\rm HFS}(e^- \mu^+)/E_F^\mu} - 1. \quad (4)$$

The large contributions from QED corrections cancel in $\Delta_{\rm HFS}$. Since the HFS of hydrogen and muonium, as well as the ratio of muon and proton magnetic moments, have been measured to better than 30 ppb, $\Delta_{\rm HFS}$ can be determined to high precision from experiment.

From Eqs. (2) and (3), we have

$$\frac{E_{\rm HFS}(e^- p)/E_F^p}{E_{\rm HFS}(e^- \mu^+)/E_F^\mu} = \frac{(1 + \Delta_{\rm QED} + \Delta_R^p + \Delta_S)}{(1 + \Delta_{\rm QED} + \Delta_R^\mu)}. \quad (5)$$

Thus we can obtain a result for the proton structure contribution in terms of quantities measurable to high precision in atomic physics:

$$\Delta_S = \Delta_{\rm HFS} + \Delta_R^\mu - \Delta_R^p + \Delta_{\rm HFS}(\Delta_{\rm QED} + \Delta_R^\mu). \quad (6)$$

The cross terms are smaller than the uncertainties in the leading terms, and here $\Delta_{\rm QED}$ can be approximated as $\alpha/2\pi$.

The proton structure contributions consist of the classic Zemach term computed from a convolution of elastic form factors and the polarization contribution from the inelastic hadronic states contributing to the spin-dependent virtual Compton scattering: $\Delta_S = \Delta_Z + \Delta_{\rm pol}$. In addition, as we discuss below, the relativistic recoil corrections of order $\alpha m_e/m_p$ are modified by the finite size of the proton. The Zemach term takes into account the finite-size correction to the proton magnetic interactions as well as the finite-size distortions of the electron's orbit in the hydrogen atom [5,6]: $\Delta_Z = -2\alpha m_e \langle r\rangle_Z (1 + \delta_Z^{\rm rad})$, where $\langle r\rangle_Z$ is the radius of the proton as calculated from the Zemach integral

$$\langle r\rangle_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[G_E(Q^2)\frac{G_M(Q^2)}{1+\kappa_p} - 1\right], \quad (7)$$

with G_E and G_M the electric and magnetic form factors of the proton, normalized with $G_E(0) = G_M(0)/(1 + \kappa_p) = 1$, and $\kappa_p = g_p/2 - 1$. Additionally, $\delta_Z^{\rm rad}$ is a radiative correction to the Zemach term estimated in [11]. It has been calculated analytically in [18] for the case where the form factors are represented by dipole forms: $\delta_Z^{\rm rad} = (\alpha/3\pi)[2\ln(\Lambda^2/m_e^2) - 4111/420]$. With $\Lambda^2 = 0.71\ {\rm GeV}^2$, this yields $\delta_Z^{\rm rad} = 0.0153$.

The main part of the inelastic contribution can be constructed from the work of Iddings [7], and Drell and Sullivan [8]. Compact expressions are given by De Rafael [9], Gnädig and Kuti [10], and Faustov and Martynenko [3] in terms of the Pauli form factor F_2 and spin-dependent structure functions g_1 and g_2 of the proton.

Evaluation of the constraint.—We will consider each term on the right hand side of Eq. (6). To compute $\Delta_{\rm HFS}$ from (4), we use the measured hydrogen HFS [19] $E_{\rm HFS}(e^- p) = 1420.405\,751\,766\,7(9)$ MHz and muonium HFS [20] $E_{\rm HFS}(e^- \mu^+) = 4463.302\,765(53)$ MHz. The measured masses are [21] $m_p = 938.272\,029(80)$ MeV, $m_\mu = 105.658\,369(9)$ MeV, and $m_e = 0.510\,998\,918(44)$ MeV. The ratio of magnetic moments has been measured to high precision, ± 0.028 ppm; the value obtained is [22] $\mu_\mu/\mu_p = 3.183\,345\,118(89)$. From these values we find $\Delta_{\rm HFS} = 145.51(4)$ ppm.

The order-α relativistic recoil correction Δ_R^N has been computed by Arnowitt [23] for muonium ($N = \mu$). Bodwin and Yennie [11] quote the corrections to second order in α in their Equation (1.10), which is analogous to Eq. (8) below. With use of [21] $\alpha^{-1} = 137.035\,999\,11(46)$ and [24] $\kappa_\mu = 0.001\,165\,920\,8(6)$, this correction is evaluated to be $\Delta_R^\mu = -177.45$ ppm.

Bodwin and Yennie [11] have also computed the corrections to their formula in the hydrogen case due to the finite size of the proton from elastic intermediate states. Note that these are finite-size corrections to the recoil correction and are distinct from the Zemach correction. A mark of the distinction is that after scaling out the lowest order Fermi HFS, the recoil corrections go to zero as $(m_p/m_e) \to \infty$, whereas the Zemach correction does not. The Bodwin-Yennie pointlike result to order α^2 is [11]

$$\Delta_R^p = \frac{\alpha}{\pi}\frac{m_e m_p}{m_p^2 - m_e^2}\left(-3 + 3\kappa_p - \frac{9}{4}\frac{\kappa_p^2}{1+\kappa_p}\right)\ln\frac{m_p}{m_e} + \alpha^2\frac{m_e}{m_p}\left\{2\ln\frac{1}{2\alpha} - 6\ln 2 + \frac{65}{18} + \kappa_p\left[\frac{7}{4}\ln\frac{1}{2\alpha} - \ln 2 + \frac{31}{36}\right] + \frac{\kappa_p}{1+\kappa_p}\left[-\frac{7}{4}\ln\frac{1}{2\alpha} + 4\ln 2 - \frac{31}{8}\right]\right\}, \quad (8)$$

with [21] $\kappa_p = 1.792\,847\,351(28)$. This gives $\Delta_R^p = (-2.01 + 0.46)$ ppm, where the two terms are from $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$. Quoting [11], finite-size corrections change this to $\Delta_R^p = [+5.22(1) + 0.46]$ ppm $= 5.68(1)$ ppm, where the quoted error is an estimate using the dipole form factor for the proton (both G_E and G_M) with mass parameter $\Lambda^2 = 0.71 \pm 0.02\ {\rm GeV}^2$. An additional correction [18] of 0.09 ppm brings Δ_R^p to 5.77 ppm.

Volotka *et al.* [25] have reevaluated the finite-size corrections to the proton recoil corrections with the same magnetic radius, but with a charge radius taken from Ref. [26], and find $\Delta_R^p = 5.86$ ppm, or 0.18 ppm larger than Bodwin and Yennie. By forcing the magnetic form factor to reproduce their result for the Zemach integral, Volotka *et al.* obtain a second value of 6.01 ppm. We shall use the first Volotka result and include an uncertainty of 0.15 ppm to cover the difference between the modified Bodwin-Yennie and the second Volotka determinations. Note that structure-dependence uncertainty within the recoil corrections is still well under the uncertainty of the polarization terms, and that this uncertainty in the recoil term can be reduced as knowledge of the form factors improves.

From the individual values for $\Delta_{\rm HFS}$, Δ_R^μ, and Δ_R^p, we obtain $\Delta_S = -37.66(16)$ ppm. Thus the contribution of proton structure is constrained by atomic physics with an uncertainty well under 1%.

The Zemach term.—We shall subtract the polarization contributions to isolate the Zemach term and then explore its relevance to new form factor parametrizations. Although the polarization contributions have been long known to be small [9,10], the error in Δ_Z is essentially all due to the uncertainty in $\Delta_{\rm pol}$. From Faustov and Martynenko [3], we take $\Delta_{\rm pol} = 1.4 \pm 0.6$ ppm, which implies $\Delta_Z = -(39.1 \pm 0.6)$ ppm and thus $\langle r \rangle_Z = (1.019 \pm 0.016)$ fm. The unit conversion used $\hbar c = 197.326\,968(17)$ MeV fm.

Predictions for Δ_Z and $\langle r \rangle_Z$ as computed from a selection of parametrizations of the form factors are given in Table I. The first row is the textbook standard, wherein both G_M and G_E are given by the dipole form. The result, $\Delta_Z = -38.72$ ppm, can already be found in [11]. New analytic fits to the form factors [27,28] make a significant change in the Zemach integral, of up to 6%. The form factor parametrization given in [26] yields [6] $\langle r \rangle_Z = 1.086(12)$ fm. It is not clear why the large difference exists. The scattering data is subject to radiative and other corrections; any difference highlights the usefulness of having the precise value that we have derived. Not all of the Δ_Z or $\langle r \rangle_Z$ for the different models in the table are compatible with the results extracted from the analysis of the atomic data. However, the G_M–G_E combination suggested in the third row from the end of the table shows that fully compatible models exist.

The table also shows results for the charge radius $\sqrt{\langle r_E^2 \rangle} = \sqrt{-6 \frac{d}{dQ^2} G_E(Q^2)|_{Q^2=0}}$. The values compare to results from Lamb-shift measurements [29] (0.871(12) fm), a continued-fraction fit to G_E [26] (0.895(18) fm), a standard empirical fit [30] (0.862(12) fm), and the 2002 Committee on Data for Science and Technology value [22] (0.8750(68) fm).

The differences among the Zemach integrals for different form factors derive mainly from the lower Q range of the integral, where the different parametrizations of G_E are less variant, but their effect on the integral is greater. This is seen in the last two columns of Table I. About 30% of the Zemach integral comes from Q above 0.8 GeV, but little of this has to do with the form factors. Recall that the numerator of the Zemach integrand is $G_E G_M/(1+\kappa_p) - 1$, and for high Q the form factors fall away, leaving the "-1." In the region above 0.8 GeV, the -1 contributes 0.314 fm.

Two fits by Arrington [27] are used in Table I, denoted A-I and A-II. Fit A-I uses only Rosenbluth data and A-II uses G_E/G_M from the polarization results [13–15]. While A-II represents the data well overall, for Q below 0.8 GeV its G_E/G_M ratio falls too quickly by nearly a factor of 2 compared to the actual polarization data. The same is true for the fit denoted JLab [14].

Discussion.—In this Letter we have shown how one can combine high-precision atomic-physics measurements of the ground-state hydrogen and muonium HFS and the ratio of muon to proton magnetic moments to isolate the proton structure contributions. In our method, the theoretically complex QED corrections to the bound-state problem do not appear [31]. Remarkably, the total proton structure contribution $\Delta_S = -37.66(16)$ ppm to the hydrogen HFS is determined to better than 1%. Since the polarization

TABLE I. Proton electric charge radius $\sqrt{\langle r_E^2 \rangle}$, Zemach contribution Δ_Z to the HFS, and Zemach radius $\langle r \rangle_Z$ for various parametrizations of G_E and G_M. The results should be compared to $\Delta_Z = -(39.1 \pm 0.6)$ ppm or $\langle r \rangle_Z = (1.019 \pm 0.016)$ fm, as obtained from analysis of atomic HFS data. The dipole form is $G_M(Q^2) = (1+\kappa_p)/(1+Q^2/0.71\ {\rm GeV}^2)^2$. The G_E labeled JLab is [14] $(1 - 0.13\frac{Q^2}{{\rm GeV}^2})\frac{G_M}{1+\kappa_p}$. Parametrizations A-I and A-II are from [27]. Those labeled Brodsky-Carlson-Hiller-Hwang (BCHH), I and II, use $F_2/F_1 = [1/\kappa_p^2 + Q^2/(1.25\ {\rm GeV})^2]^{-1/2}$ and $F_2/F_1 = \kappa_p[1 + (Q^2/0.791\ {\rm GeV}^2)^2 \ln^{7.1}(1 + Q^2/4m_\pi^2)]/[1 + (Q^2/0.380\ {\rm GeV}^2)^3 \ln^{5.1}(1 + Q^2/4m_\pi^2)]$, respectively, [28]. The last column gives the contribution to $\langle r \rangle_Z$ from $Q > 0.8$ GeV.

Parametrizations G_M	G_E	$\sqrt{\langle r_E^2 \rangle}$ (fm)	Δ_Z (ppm)	$\langle r \rangle_Z$ (fm) total	$Q > 0.8$ GeV
dipole	$G_M/(1+\kappa_p)$	0.811	-38.72	1.009	0.305
dipole	JLab	0.830	-39.23	1.022	0.306
A-I	A-I	0.868	-40.84	1.064	0.305
A-I	$G_M/(1+\kappa_p)$	0.863	-40.71	1.061	0.305
A-II	A-II	0.829	-39.68	1.034	0.305
A-II	JLab	0.855	-40.09	1.045	0.305
dipole	BCHH-I	0.789	-38.26	0.997	0.305
A-II	BCHH-I	0.905	-39.12	1.019	0.305
dipole	BCHH-II	0.881	-40.29	1.050	0.305
A-II	BCHH-II	0.905	-41.14	1.072	0.305

contribution can be determined from the measured spin-dependent proton structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$, we obtain a precise value for the Zemach radius $\langle r\rangle_Z = (1.019 \pm 0.016)$ fm, which is defined from a convolution of the G_E and G_M form factors. This new determination gives an important constraint on the analytic form and fits to the proton form factors at small Q^2. The precision of the Zemach radius will be further improved when new, more precise data for g_1 and g_2, especially at small ν and Q^2, becomes available.

The proton structure terms can also be extracted using the hydrogen HFS alone [25,32]. The Zemach radius obtained this way is larger than our result; however, this determination depends on the QED bound-state radiative correction.

Conversely, by combining the muonium and hydrogen HFS data, one can obtain an "experimental" value for the purely QED bound-state radiative corrections: $\Delta_{\rm QED} = 1135.27(13)$ ppm. This gives $\Delta_{\rm QED} - \alpha/2\pi = -26.14(13)$ ppm. To minimize the uncertainty, we take advantage of the measured ratio [22] $m_p/m_e = 1836.152\,672\,61(85)$. This value of $\Delta_{\rm QED}$ is approximately 0.9 ppm smaller than the calculated QED correction used in [25,32]. Neither the uncertainty in the polarization nor nuclear recoil corrections contribute to this difference. It is worth noting that in the case of the positronium HFS, the theoretical prediction from the QED bound-state radiative corrections is also higher than the experimental value by several standard deviations [33].

Our method of combining experimental atomic physics has other applications; for example, measurements of the difference of the Lamb shifts (or Rydberg spectra) of muonium and hydrogen could potentially give a very precise value for the proton's electric charge radius, since again the QED radiative corrections essentially cancel. Similarly, the difference of lepton anomalous moments $a_\mu - a_e$ directly exposes the hadronic and weak corrections to the muon moment.

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