PH 585
AMO

Lecture # 8

Fine structure splitting

Energy corrections due to $H'_{1,2,3}$ $\Rightarrow$

$$\Delta E_1 \overset{\uparrow}{=} -E_n \frac{(Z\alpha)^2}{n^2}\left[\frac{3}{4} - \frac{n}{l+\frac{1}{2}}\right]$$

HW!

$$\Delta E_2 = -E_n \frac{(Z\alpha)^2}{2nl(l+\frac{1}{2})(l+1)} \times \begin{cases} l & \text{for } j = l+\frac{1}{2} \\ -l-1 & \text{for } j = l-\frac{1}{2} \end{cases}$$

$\Delta E_2 = 0$ for $l=0$

$$\Delta E_3 = -E_n \frac{(Z\alpha)^2}{n} \text{ for } l=0 \text{ and } 0 \text{ for } l \neq 0$$

Total fine-structure correction:

HW!

$$\Delta E = \Delta E_1 + \Delta E_2 + \Delta E_3 \overset{\downarrow}{=} \underbrace{E_n \frac{(Z\alpha)^2}{n^2}\left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4}\right)}$$

Total energy:

$$E_{nj} = \underbrace{E_n \left[1 + \frac{(Z\alpha)^2}{n^2}\left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4}\right)\right]}$$

Note: dependence on j, and not on l!

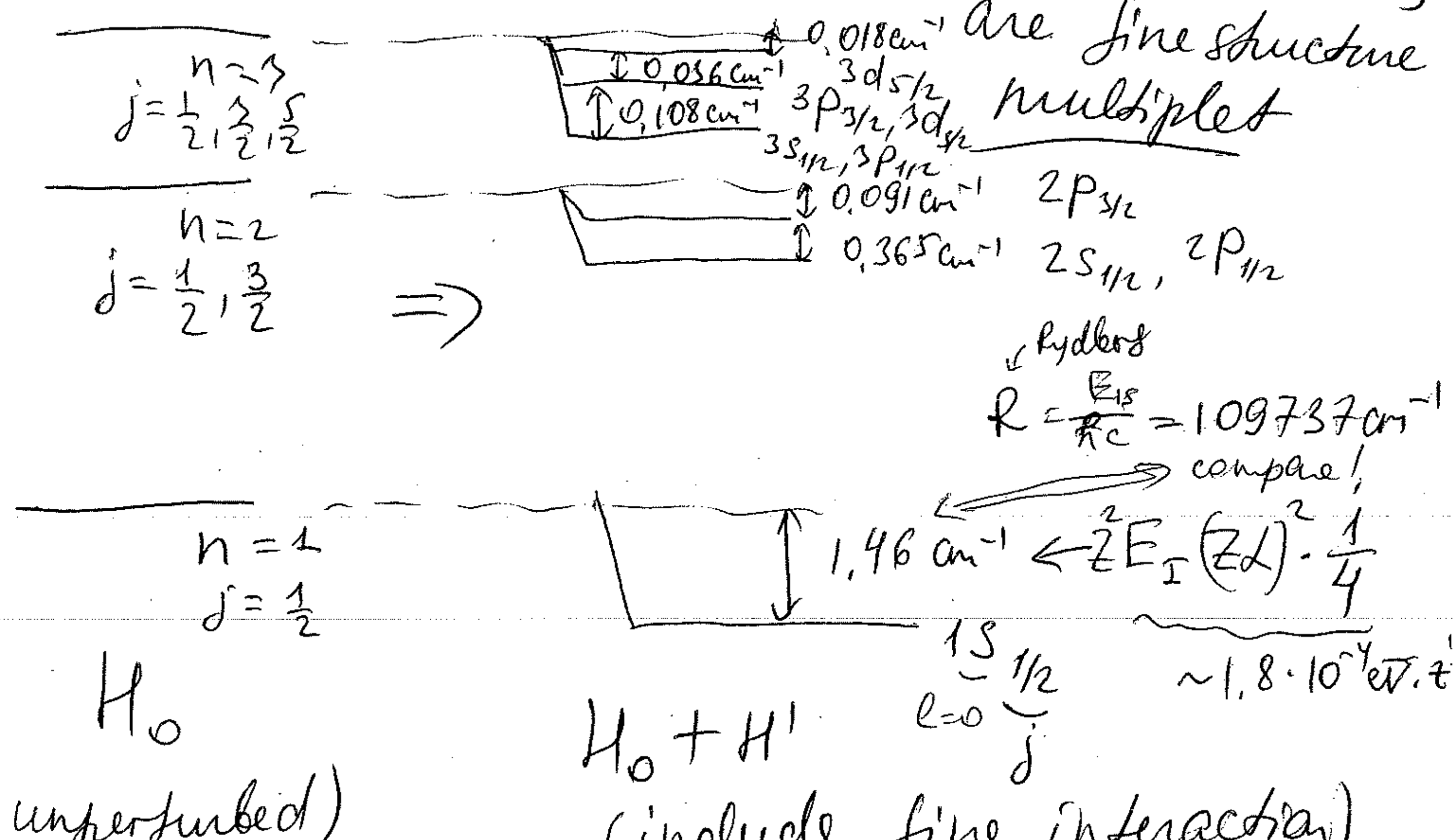
$$E_n = -\frac{E_I Z^2}{n^2}$$

So, $2n^2$ degeneracy of the n^{th} level is partially removed $\Rightarrow$ for n fixed $\Rightarrow l = 0, \dots, n-1$

$$j = \frac{1}{2}, \frac{3}{2}, \dots, \underbrace{n - \frac{1}{2}}$$

$$|l - s| \le j \le |l + s|$$

these n levels



<u>Note</u>: difference in energy between $j = \frac{1}{2}$ and

$$j = n - \frac{1}{2} \Rightarrow \delta E = \underbrace{|E_n|}_{\frac{Z^2 E_I}{n^2}} (Z\alpha)^2 \frac{n-1}{n^2} = \frac{Z^4 \alpha^2 (n-1)}{2n^4} E_I$$

for any $n \neq 1$ level

splitting

As $n \uparrow \Rightarrow \delta E \downarrow$

$l = 0 \Rightarrow$ shift

other levels $\Rightarrow$ splitting

<u>Note</u>: Z^4-dependence of δE!

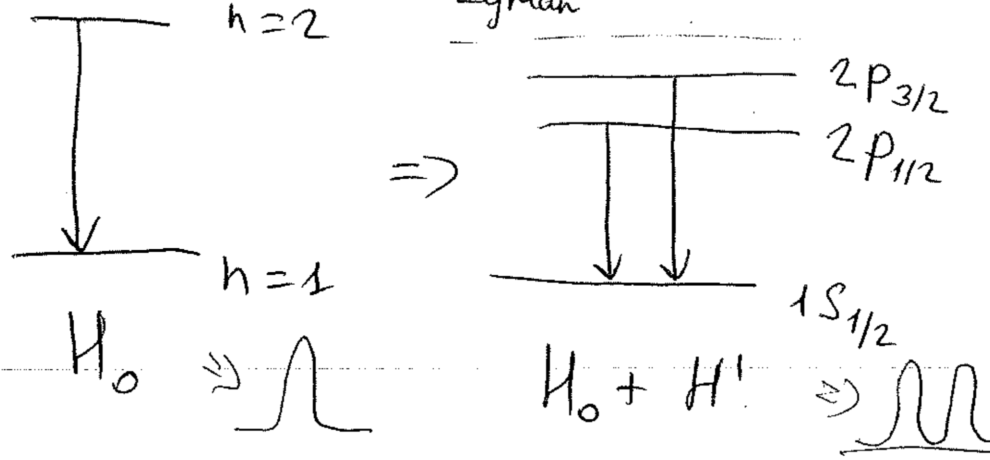
Fine structure of spectral lines

Consider transitions allowed in electric dipole approx. $\Rightarrow \Delta l = \pm 1 \Rightarrow \Delta j = \pm 1, 0$

$\vec{D} = -e\vec{r}$ (doesn't couple spin)

So, e.g., $Ly\alpha$ line of (H)-atom $\Rightarrow$

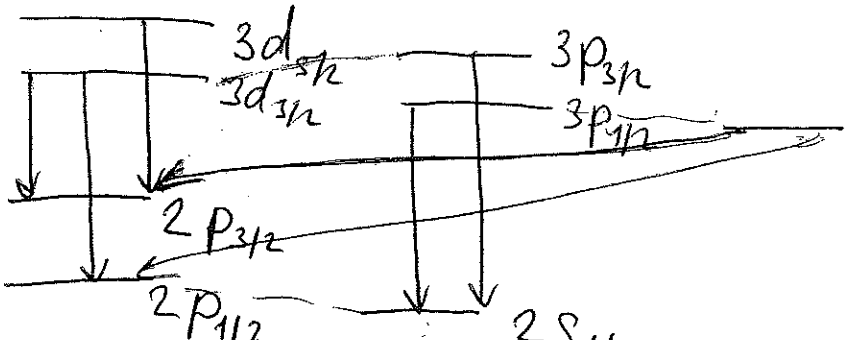
Lyman



$\Rightarrow$ ~~lower~~ different frequency and splits into two lines $\Rightarrow$ doublet

$H_\alpha \leftarrow$ α-line of Balmer series of (H) atom $\Rightarrow$

656.3 nm



7 transitions, but since

$3d_{3/2} \rightarrow 2p_{1/2}$ and

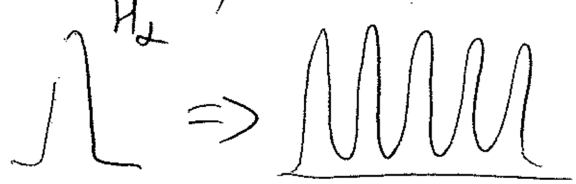
$3p_{3/2} \rightarrow 2S_{1/2}$,

as well as

$3P_{1/2} \rightarrow 2S_{1/2}$ and

$3S_{1/2} \rightarrow 2P_{1/2}$ have same frequencies (in this approx.)

So,



5 lines $\Leftarrow$

B&J

see p. 248 for relative intensities

(4)

Since δE $\downarrow$ as $n \uparrow$ $\Rightarrow$ fine structure splitting is mostly due to the lower level splitting $\Rightarrow$

$\uparrow$ splitting

in Balmer series $\Rightarrow$ doublets $\Rightarrow$

$$\left.\begin{array}{l} 3d_{3/2} \to 2p_{1/2} \\ 3s_{1/2} \to 2p_{1/2} \\ 3p_{1/2} \to 2s_{1/2} \end{array}\right) \ (1)$$

$= 3p_{1/2} \to 2s_{1/2}$

e.g. for $n=3 \to n=2$ $\Rightarrow$

$3s_{1/2} \to 2p_{1/2} =$

in Pashen series

$\Downarrow$

three groups of lines

etc

$$\left.\begin{array}{l} 3d_{5/2} \to 2p_{3/2} \\ 3d_{3/2} \to 2p_{3/2} \\ 3s_{1/2} \to 2p_{3/2} \end{array}\right) \ (2)$$

Relative intensities $\Rightarrow$ ~~[illegible]~~

~~[illegible]~~ Consider a specific example

~~shouldn't depend on m $\Rightarrow$ take m = m' = 0 and r' = [illegible]~~

$\Downarrow$

Compare line intensities for $np_{3/2} \to n's_{1/2}$ and $np_{1/2} \to n's_{1/2}$ transitions

Need $\Rightarrow$ $|n l m_l \underbrace{m_s}_{=\pm\frac{1}{2}}\rangle \Leftrightarrow |n l \underbrace{j}_{=l\pm\frac{1}{2}} \overbrace{m_j}^{-j<m_j<j}\rangle$

Recall:

$$|\underbrace{l \pm \tfrac{1}{2}}_{j}, m_j\rangle = \frac{1}{\sqrt{2l+1}}\left(\pm\sqrt{l \pm m_j + \tfrac{1}{2}}\ |l, \underbrace{m_j - \tfrac{1}{2}}_{m_l}, \underbrace{\tfrac{1}{2}}_{m_s}\rangle + \right.$$

Sakurai p. 215

$+\cancel{\ell}\sqrt{\ell \mp m_j + \frac{1}{2}}\, |\ell, \underbrace{m_j + \tfrac{1}{2}}_{m_\ell}, \underbrace{-\tfrac{1}{2}}_{m_s}\rangle)$

Then $\Rightarrow$ need $|\vec{r}_{ba}|^2 = |\langle \underbrace{n'\ell' j' m_j'}\, |\vec{r}|\, \underbrace{n\ell j m_j}\rangle|^2$

$\sum_{m_\ell, m_s} A_{\ell s} |n \ell m_\ell m_s\rangle$

Then, calculate all possible $\langle n'\ell' m_\ell' m_s' |\vec{r}| n \ell m_\ell m_s\rangle$

The rest is similar to Problem #2 of Homework 4

$$\frac{I_{p_{3/2} \to s_{1/2}}}{I_{np_{1/2} \to n's_{1/2}}} = \frac{\hbar \omega_{3/2 \to 1/2}}{\hbar \omega_{1/2 \to 1/2}} \underbrace{\frac{W^D_{3/2 \to 1/2}}{W^D_{1/2 \to 1/2}}}_{} =$$

$$= \frac{\omega^4_{3/2 \to 1/2}}{\omega^4_{3/2 \to 1/2}} \frac{|r_{3/2 \to 1/2}|^2}{|r_{1/2 \to 1/2}|^2}$$

$\Rightarrow \frac{2m}{3\hbar} \omega_{1/2 \to 1/2} |r_{1/2 \to 1/2}|^2 = |f_{1/2 \to 1/2}|$

$W^D_{1/2 \to 1/2} = \frac{2\hbar\alpha}{mc^2} \omega^2_{1/2 \to 1/2} |f_{1/2 \to 1/2}|$

For $\cancel{s}$ sp transitions: $\frac{I(S_{1/2} \to P_{3/2})}{I(S_{1/2} \to P_{1/2})} = \frac{2}{1}$

pd transition: $\frac{I(p_{3/2} \to d_{5/2})}{I(p_{3/2} \to d_{3/2})} = \frac{9}{1}$

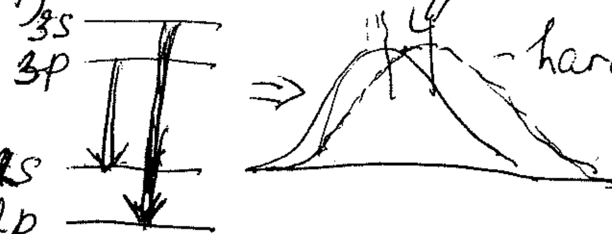
see p. 248 for more

The Lamb shift

Dirac theory $\Rightarrow E_{n,j} \Rightarrow$ so, i.e., $2S_{1/2}$ and $2P_{1/2}$ $\Rightarrow$ same energy

Experiment: $2S_{1/2}$, $2P_{1/2}$ — $0.03\ \mathrm{cm}^{-1}$ $\to$ tiny! $\to$ hard to measure due to Doppler broadening of lines

Houston (1937)
Williams (1938)
Lamb & Retherford (1947)] – decisive
see PRB 72, 241 (1947)



$\Rightarrow$ hard to resolve optical spectra

$\Downarrow$

use microwave tech. to directly probe $P_{3/2}$, $P_{1/2}$

Rf $\leftarrow$ radio frequencies

Ex.: $2P_{3/2}$, $2S_{1/2}$

$\lambda = 2{,}74\ \mathrm{cm}$

$\Delta E = 0{,}365\ \mathrm{cm}^{-1}$

$\nu = 10949\ \mathrm{MHz}$

Recall Doppler broadening:

$$(\omega_1 - \omega_0)^2 = \frac{2k_BT}{Mc^2}\,\omega_0^2 \ln 2 \sim 10^{10}\ \mathrm{Hz} \Leftarrow \nu$$

$\Rightarrow \ll$ optical freq. $(\sim 10^{14-15})$

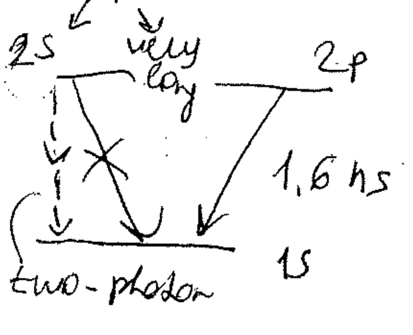
If $\omega_0 \downarrow \Rightarrow \omega_1 - \omega_0 \downarrow \Rightarrow$ reduced broadening at lower frequencies

But: since $W_{ab}^{s} \sim \nu^3 \Rightarrow$ small transition rates $\uparrow$

$\Downarrow$

need stimulated emission and unequal population of levels

$\tau = \frac{1}{7}\,s$ — very long



$\Rightarrow \tau_{2s} \gg \tau_{2p}$

metastable level

Lamb & Retherford ⇒ $E_{n=2} - E_{n=1}$ (7)

$\boxed{H_2}$ ⇒ $\boxed{H}$ (64%) ⇒ e^- @ $\gtrsim 10.2\,eV$ ⇒ $\boxed{rf}$ $\xrightarrow{2S_{1/2}}$ 1S detector

T = 2500K

(H)-atoms in $1S_{1/2}$

1 in 10^8 atoms is in $2S_{1/2}$, $2P_{1/2}$, $2P_{3/2}$

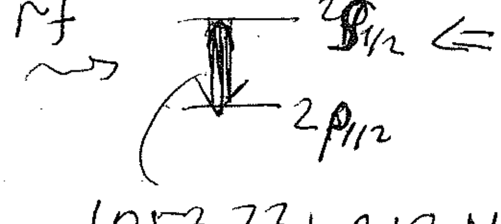
$v = 8 \cdot 10^5 \frac{cm}{s}$

detector metal (W)

atoms in 2p already decayed

$2S_{1/2} \to 1S$ transition excites or ejects, electrons from the metal

smaller signal if some of the RF gets ~~stimulated~~ (emitted) due to $2S_{1/2} \to 2P_{1/2}$, $2P_{3/2}$ transitions



1057.77 ± 0.10 MHz

How do include this into Dirac theory? ⇒

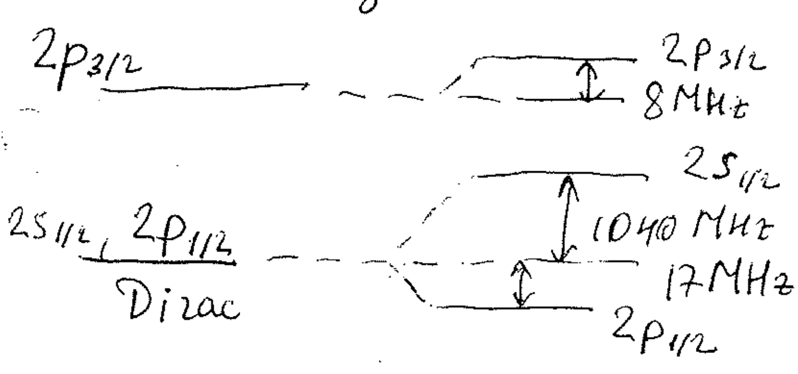
"radiative corrections" → QED → take into account interaction of the electron with the quantized EM field

$$H_{EM} = \hbar\omega \sum_k \left(a_k^+ a_k + \tfrac{1}{2}\right)$$

rapid oscillat. motion ⇐ act on e^- ⇐ fluctuations of this field even in vacuum ⇐ zero-point oscillations even at zero field

⇓

acts stronger on s-states (recall Darwin term!)



In 1972 ⇒ measured optically using Doppler-free spectroscopy of H_α line.

B&J 15.2